An agent-based model of democratic diffusion

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Abstract

Contemporary rhetoric surrounding international developments as well as recent developments in the academic literature suggest increasing attention to possible contagious effects of democratization. This paper presents one possible way to look at such contagion, by employing an agent-based model of democratic diffusion through public attitudes. Some tentative conclusions are drawn as to the conditions under which democratic diffusion is most likely to show patterns similar to trends, waves, and geographical clusters as observed in empirical data.

1 Introduction

With the ousting of Saddam Hussein by the Coalition Forces, an important argument made in public statements was that with the removal of this feared dictator, a beacon of democracy could be established in the Middle East. This beacon would enlighten the Arabs and teach the whole region the joys of democratic life. Arabs would read about the developments in their neighboring country, they would see the images on the news, and they would start to understand how democracy could work in an Arab country. It would make them see that it is possible in their country too, and it would emancipate them and make them oppose their own autocratic regimes. In other words, establishing one democracy in the Middle East would surely lead to a contagious effect, spreading the presence of democracy in the Middle East.

The democracy in Iraq may well be too young to determine whether or not such contagious effect will take place. While initial developments seem to show as much resistance to this strengthened Western influence in the Middle East as enthusiasm for more democracy, it is clear that for policy makers this idea of a diffusion of democracy is a very real thing. Especially since the collapse of the Soviet Union and the sudden wave of democratization

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in Eastern Europe, it has become common wisdom to consider democratization a contagious phenomenon. This paper will attempt to shed further light on this subject of the diffusion of democracy.

Not only policy makers have this vision of a spreading of democracy, also empirical research in political science and international relations shows that the regional context matters in democratization. When controlling for other explanations of democratization, most importantly the economic development thesis, the fact that geographically contiguous countries are democratic is still a significant factor in whether or not a country democratizes or not. Countries surrounded by democracies have a higher chance of democratizing; the democratization of the world occurs in temporal waves; and whole regions follow each other in their democratization in short periods of time, most visibly in Eastern Europe and at times in Latin America - all indicators of a process of diffusion or contagion of democracy.

Geographical patterns of democratization are patterns at a macro-level. It is the democratization of countries as a whole, in their international region, that shows these patterns. Democratization, however, to state the obvious, is in the end a micro-level process. It is individuals that alter constitutions, decide to organize elections, decide to protest against their regime, or decide whether or not to suppress the opposition. A proper understanding of the macro-level patterns of democratization cannot do without a proper understanding of these individual behavioral patterns. This linkage of macro- and micro-level patterns has always been notoriously difficult in the social sciences and by far most studies of social behavior focus on either of the two levels. One attempt to deal with this linkage has been the relatively recent introduction of agent-based modeling in the social sciences. Although early applications exist, the real popularity of agent-based modeling has only come about with the recent increase in easily accessible computing power.

In this paper an agent-based model of the diffusion of democracy is being developed where the focus is indeed on this linkage between individual behavior and global geographical dynamics. On the basis of existing models of public opinion dynamics and the role of private and public opinions in popular protests, a model is developed that, while keeping regime transitions not caused by popular protest exogenous, describes the relation between these transitions, their effect on public opinion in neighboring areas, and subsequent popular regime transitions. Studying the different combinations of model parameters, the model is studied in relation to empirical observations.

The key question in this paper is thus: *To what extent can a computational model of the diffusion of democracy through public opinion dynamics tell us under what conditions such diffusion leads to the increase in democracy, its temporal waves, and its clustering as can be observed in empirical data?*

In the remainder of this paper first, to put the simulation in proper perspective, some attention is paid to the empirical observations that are being modeled. Subsequently, a description of the agent-based model will be provided. Finally, the results of the computer simulation will be discussed.

2 Empirical observations

Before turning to the model itself, it is necessary to see exactly what patterns we are trying to model. In this section first some attention will be paid to the definition and measurement of democracy and democratization, after which visualizations of the empirical data and references to existing studies on international factors on democratization will shed some

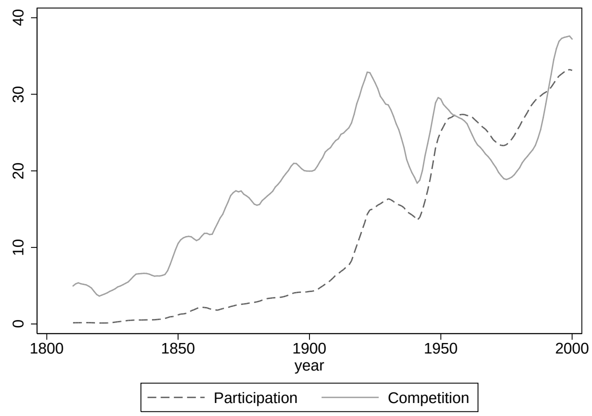

Figure 1: Vanhanen's Participation Score and Competition Score, 1810-2000

further light on exactly what patterns this paper sets out to study. Most studies start with theory, distil from this the hypotheses, and have the empirical data come in at the end to validate the theory, but in agent-based modeling there is a much more explicit attempt to replicate empirical data in the lab of a computer simulation, and thus a solid understanding of the empirical data is the point of departure rather than solely a validating factor.

2.1 Trends in democratization

The agent-based model to be presented in this paper attempts to model the diffusion of democracy through popular opinion and to use this model to provide a possible explanation for the global trends in democratization that are observed. To compare the simulation output to empirical data, a measure of the level of democracy in the real world is needed. This paper adheres to minimal definitions of democracy similar to the famous definition by Schumpeter (1976: 269) and accepts the notion of Dahl (1971) that democracy consists of two primary dimensions - competition and participation. By creating measures of democracy that take both dimensions into account, however, studies of transitions lead to a conflation of transitions that are very different in nature. When the United States extended its suffrage to blacks in 1965 or Switzerland to women in 1971, these were very different transitions from the collapse of the Soviet Union or the end of Franco's reign in Spain in November 1975. Perhaps any transition to democracy is unique, but the extension of suffrage is particularly different from the introduction of political competition. In other words, in Dahl's two-dimensional space of regime types, movements on the axis of political competition are very different from movements on the axis of political participation and therefore require very different theoretical explanations. This paper will focus solely on the level of competition.

On the basis of the work of Dahl, Vanhanen (1997) developed a measure of democracy which measures the level of participation and competition separately and then combines them in a scale of democracy. He collected this data for all countries in the world from 1810 to 2000 and made this available online. His indicators of participation and competition are unusually straightforward, respectively measuring the percentage of the overall population that turned out in an election as an indicator of the level of participation and the percentage of the vote acquired by the largest party as an indicator of the lack of competition. Many question marks can easily be placed by these indicators casting doubt on the extent to which these are proxies for the complex concepts of participation and competition, but the interesting advantage of these measures is their transparency. The fact that there is at times a lack of correspondence between these indicators and the concepts being measured is clear, but the extent to which this is the case can easily be contemplated, while with more complex measurements, like the Freedom House indicators or the Polity data set (Jaggers and Gurr 1995), this lack of correspondence becomes far more opaque. Figure 1 shows the trends of Vanhanen's two indicators over the previous two centuries.[1] The first trend that can be seen as an indication of a process of diffusion is immediately visible from those graphs: more and more countries are democratic, from less than 5 percent in 1810 to more than 30 percent in 2000.

2.2 Waves of democracy

The second striking trend in figure 1 is the pattern of waves visible in the graph. The number of competitive democracies increases to a peak in 1923, down to only 18 percent in 1943, to a new peak in 1957, a return to 18 percent in the 1970s and finally a peak just before the turn of the century of 39 percent. These waves, brought to everyone's attention by Huntington (1991), are a further indication that the international context of democratization matters. The waves seem to indicate a kind of *Zeitgeist* or ripple effect in the international environment of regimes. Either international factors similar for a set of countries or a kind of contagious or demonstrating effect must be taking place for such waves to form. Although the *Zeitgeist* perspective might be most consistent with natural understandings of waves, with global trends towards democratization which are almost inevitably followed by reverse waves, the definition of Huntington refers rather to periods where transitions to democracy simply outnumber those in the other direction. As Kurzman (1998) points out, a simple count of the number of transitions to and from democracy over time demonstrates how the waves are not at all visible in this sense. Figure 2 demonstrates this effect, where an overview of the average size of positive and negative transitions only shows the waves to a limited extent.[2] There are not so much periods of democratization of the world, but rather periods in which democratizations happen to outnumber collapses of democracies and vice versa. Whereas this idea of a *Zeitgeist* focuses on global developments, the pattern of waves can also be explained by more local relations between countries. When a regime transition in one country has a ripple effect on neighboring or otherwise linked countries, this can create global patterns of waves as described by Huntington and others (Starr 1991; Kurzman 1998; Doorenspleet 2001; Gleditsch 2002). Such local effects of democratic waves would cause a geographical clustering of political regimes.

2.3 Clustering of democracy

The third empirical observation in relation to the diffusion of democracy is that of its geographical clustering. Eye-balling over a map shows clear groups of geographically contiguous countries that are either all democratic or all non-democratic. The human capacity to see patterns where there are none is significant, however, and it is also physically im-

[1] The values have been smoothened by a moving average over seven years.

[2] The trends in figure 2 are smoothened with a moving average over seven years.

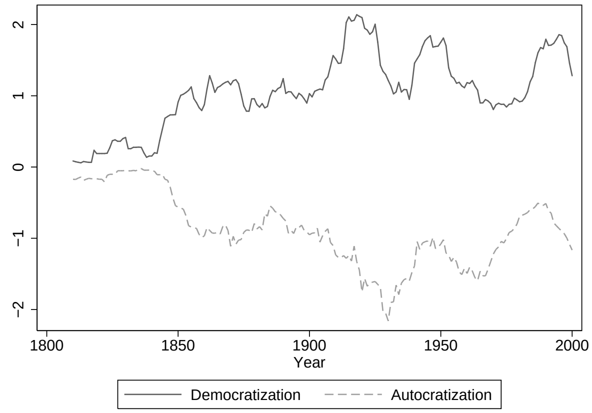

Figure 2: Average Size of Positive and Negative Transitions in Vanhanen's Competition Score, 1811-2000

possible to include worldwide maps for each time period in this paper, hence a statistic capturing the global level of clustering is required. For this the standard measure for spatial autocorrelation, Moran's I, originally developed for autocorrelation in time series, is used (Moran 1947, 1948, 1950; Anselin 1988; Gallo 2000; Gleditsch 2002).

To be able to measure the level of clustering, the first step has to be to define a matrix defining which units in the data set are adjacent to which other units. In the remainder of this section a square contiguity matrix W_t of dimension n is used, which is defined as follows:

$$W_{ijt} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are contiguous at time } t \\ 0 & \text{otherwise} \end{cases} \quad \text{where} \quad w_{iit} = 0 \quad \forall \quad i, t \tag{1}$$

With $i = \{1, 2, \cdots, n\}$ and $j = \{1, 2, \cdots, n\}$ indicating the units. Subsequently this matrix is standardized for each year such that the rows all add up to one. The standardized matrix will be denoted as $\tilde{W}_{ijt}$. For this section the contiguity data has been based on the Correlates of War project, using the Direct Contiguity data set (version 3.0), which includes land borders and overseas neighbors for different distances for countries worldwide between 1816 and 2000 (Stinnett et al. 2002). Only land borders have been taken into account.

The formula for Moran's I, as presented by Gallo (2000), is as follows:

$$I_t = \frac{\sum_i \sum_j \tilde{w}_{ijt}(x_{it} - \overline{x}_t)(x_{jt} - \overline{x}_t)}{\sum_i \sum_j \tilde{w}_{ijt}} \cdot \frac{N_t}{\sum_i (x_{it} - \overline{x}_t)^2} \quad \text{where} \quad \overline{x}_t = \frac{1}{N_t} \sum_i x_{it} \tag{2}$$

With $\tilde{w}_{ijt}$ being the standardized value of the contiguity matrix W_t, x_{it} being the respective score for unit i in year t, and N_t being the number of observations for year t. Plotting these scores over time, one gets an impression of the level of global clustering over time. The left hand side of this equation can be interpreted as the average of the covariances of all contiguous locations, while the right hand side divides this value by the average level of variance to standardize the result. In a situation where contiguous units tend to be different from each other, one of the two having a value on x below the average and the

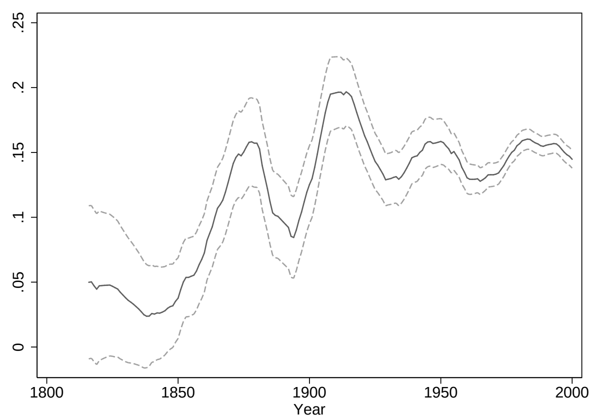

Figure 3: Moran's I on Vanhanen's Competition Score, 1816-2000, with 95% confidence intervals

other a value above average, similar to a chess board, Moran's I will be negative. Moran's I is positive in the opposite case, where contiguous units tend to be similar, either both above or both under the average. When Moran's I does not significantly differ from zero, the pattern does not differ from a random distribution of units, neither in the sense of being clustered nor in the sense of not being clustered in a structured way. Under the normality assumption, the standard error of this statistic can be calculated as follows (Gleditsch 2002; Cliff and Ord 1973):

$$E(\sigma^2_{I_t}) = \frac{N_t^2 S_1 - N_t S_2 + 3S_0^2}{S_0^2(N_t^2 - 1)} \tag{3}$$

where

$$S_0 = \sum_i \sum_j (w_{ijt} + w_{jit}), S_1 = \frac{1}{2} \sum_i \sum_j (w_{ijt} + w_{jit})^2, S_2 = \sum_i \sum_j (\tilde{w}_{ijt} + \tilde{w}_{jit})^2 \tag{4}$$

Since in the analysis presented here w_{ijt} can only have the values 1 or 0, and W_t is a symmetric matrix, this can be reduced to

$$S_0 = S_1 = 2 \sum_i \sum_j w_{ijt}, S_2 = \sum_i \sum_j (\tilde{w}_{ijt} + \tilde{w}_{jit})^2 \tag{5}$$

Figure 3 shows the level of clustering of Vanhanen's competition score over time. The level of clustering is clearly relatively volatile over time, with waves of higher levels of clustering and periods where Moran's I does not significantly differ from zero. The dotted lines denote the standard error of the measure and decreases over time due to the increase in the number of countries. The initially low levels of clustering can be explained by the low number of democracies, which means a relatively random scattering of those few democracies around the world. It appears that geographic diffusion does not so much explain the very first democracies, but clustering does become present in later transitions. Towards the end, the level of clustering becomes relatively more stable and stays consistently significantly different from zero, but the latter is likely to be partly due to the increase in the number of countries, from 21 in 1816, to 184 in 2000.[3]

It should be noted that clustering of democracy also implies clustering of autocracies. The international rise in democracy has lead to a considerable level of optimism among scholars about the future of democracy and has resulted in a strict focus on processes of democratization as opposed to autocratization. The danger of this focus is a certain bias in interpretations, for example visible in the apparently unfounded optimism concerning Russian democracy during the previous decade. Despite the danger of this bias, however, this paper will concentrate primarily, especially in its theoretical foundations, on processes of democratization. In the model developed below, the diffusion of democracy will be stronger than that of autocracy.

3 An agent-based model

In the model that forms the core of this paper, the diffusion of democracy is explained at the level of individual communication among citizens. The key to the model is the link between assumptions concerning the behavior of individual citizens in the model and the global patterns of democratization that emerge. Under what conditions does a diffusion of attitudes towards the regime indeed result in a global increase, clustering, and wave pattern of democratization as the data show us? The model is well embedded in the literature on the diffusion of opinions and the cascading effect observed in revolutions. These two theoretical approaches will be discussed here before turning to the implementation and details of the model itself.

The mechanism by which public opinion leads to revolution and an overthrow of the existing political regime is based on the idea of cascading revolutions (Granovetter 1978; Kuran 1991, 1995; Lohmann 1994). Studied mainly in the context of developments in Eastern Europe in the 1990s, and in particular the former German Democratic Republic, this model describes how what seems like a very stable regime can suddenly collapse due to a hitherto hidden public opinion among the population. When a citizen is opposed to the regime, but sees everybody else agreeing, this citizen is likely to be too scared to make his or her opinion public, fearing the repercussions in terms of government repression or social marginalization. The more opposed to the regime, however, the more likely this person is to nonetheless initiate a protest and turn to the streets. Kuran described this mechanism as having private truths - knowing that you are opposed to the regime - while having public lies - acting as if you are supporting the regime to avoid repression or social exclusion (Kuran 1995). This phenomenon has also been observed extensively in German opinion poll research, where predictions of election outcomes could be predicted more accurately when looking at how respondents think fellow citizens will vote than when looking at how those respondents think they will vote themselves. The 'spiral of silence' leads people who think they are in the minority to hide their preferences, strengthening the support for the perceived majority opinion (Noelle-Neumann 1993).

When the size of the protest increases, however, the threshold to join gradually becomes lower. Those that are opposed to the regime, but less daring, will join the protest when enough people are already on the streets, since the law of large numbers reduces the risks

[3] A clear breakpoint in the trend is visible in 1943, when the number of countries is 53, after which the rate of growth of the number of countries increases dramatically. The numbers presented are the number of observations per year in the Vanhanen dataset.

of government oppression as well as of marginalization. A few more protesters in the streets can thus ignite a cascade of protest with more and more people daring to voice their opinions truly. After a while it will become clear that the position of the regime becomes untenable and supporters of the regime start to become concerned with their future position. They might be in favor of the regime, but if they do not protest, they risk social marginalization after the government has been overthrown. In the end, even supporters of the regime join the protests, professing public lies to protect their position in the society after the revolution. The monthly protests in Leipzig were a good example of this cascading model of revolution, where every month more protesters joined as people saw the lack of repression and the size of existing support (Lohmann 1994).

Whereas the model of cascading revolutions sheds light on how attitudes and opinions lead to protests or other forms of explicit expression of these attitudes, the social judgment theory contributes to the modeling of attitude change in an agent-based model (Jager and Amblard 2005). The basic premise of the theory is straightforward: "Whereas the quality of arguments may determine the extend to which one is being persuaded by another person, often people respond quite simply by favoring positions close to their own, and rejecting more distant positions" (Jager and Amblard 2005: 295). In the formalization of the model as proposed for agent-based models by Jager and Amblard, two threshold values are of importance. When the opinion of a fellow agent, j, is within what they call the latitude of acceptance for individual i, u_i, i will update his or her opinion towards that of j. On the other hand, when the opinion of j is beyond the latitude of rejection for i, t_i, i will update away from j. When the attitude of j is in between those two thresholds for i, i will not update his or her opinion at all. Formally, the following rules are applied when two agents, i and j communicate:

$$\delta x_i = \begin{cases} \mu(x_j - x_i) & \text{if} \quad |x_i - x_j| < u_i \\ \mu(x_i - x_j) & \text{if} \quad |x_i - x_j| > t_i, \end{cases} \tag{6}$$

where $t_i > u_i$, with μ controlling the strength of the effect (Jager and Amblard 2005: 296). In their simulations, depending on the values for u_i and t_i (in their model always the same for all i), either two extreme opinions, one average opinion, or a group of clear clusters emerge. One simple and general model can thus lead to uniformity, bipolarization, and pluriformity in an agent-based setting of opinion diffusion.

4 The model implementation

In the description of the model, it is useful to make a distinction between two different stages: the setup of the model and the different iterations. At the start of each simulation, agents are placed on the board and initial settings are assigned. Once the model is all initialized, it will iterate over a large number of iterations, whereby every iteration similar steps are taken. These initial settings, and these iterative steps, are discussed in this section.

4.1 The setup

In the setup stage three types of agents are created and initialized: the provinces, countries, and citizens. Let us first turn to the creation and placement of the provinces. The provinces form the cells of what is commonly known as cellular automata of size W by

H.[4] Cellular automata are a grid of adjacent square cells which keep changing state using simple rules and on the basis of information from the previous state of the cell and the state of cells in what is called the Neumann neighborhood, the four cells directly adjacent. The most famous example of a cellular automata is the Game of Life, which is a small set of very simple rules which leads to cyclic patterns, *perpetua mobilia*, and patterns far more complex than expected from the initial rules (Gardner 1983). Although cellular automata form the basic foundation and inspiration for this model, in line with a model of democratic diffusion by Cederman and Gleditsch (2004), the model does deviate on several fronts from regular cellular automata models. One aspect that is unusual for cellular automata is that the map of this model wraps around the borders. The cells at the edges of the map are directly adjacent to those on the opposite edge - similar to creating cellular automata on the surface of a torus. This is common in computer games that are based on cells and that try to simulate the fact that the world is round.

Once the provinces have been created country borders are added to the map. Drawing borders grouping together certain provinces is of course a clear deviation from any common cellular automata. The country borders are created by an algorithm where countries "conquer" neighboring provinces which become part of the country of the conquering province, unless this leads to a fragmented country that the province is originally from. $W \times H \times M$ times a random combination of two neighbouring provinces, P_1 and P_2, is selected. If $C_{P_1} \neq C_{P_2}$, C_{P_1} will conquer P_2, unless this leads to a disconnected former C_{P_2}.[5] This algorithm results in a somewhat realistic looking map, with varying forms and sizes of countries, and is derived from the model of Cederman and Gleditsch (2004): their model models wars between countries, whereas in this model these wars are only applied to the setup stage to create the borders. Each country C is subsequently assigned a random level of isolation, $\varphi_0 \sim N_{[0,100]}(\phi_{mean}, \phi_{std})$.[6] With a probability π, the country is set to be a democracy ($\Omega = 1$), otherwise it is set to be an autocracy ($\Omega = 0$). Randomly one of the provinces of the country is assigned as the capital.

For each province a random number of citizens is set, $N_{citizens} \sim N_{[1,\infty)}(C_{mean}, C_{std})$, with an initial attitude towards democracy $\alpha_i \sim N_{[0,\lambda-1]}(A_{mean}, A_{std})$. The threshold values of the social judgment model are assigned as $t_i \sim N_{[0,\infty)}(T_{mean}, T_{std})$ and $u_i \sim N_{[0,t_i]}(U_{mean}, U_{std})$. By default, a citizen is not protesting, $\psi_i = 0$.

4.2 The iteration

Once all initial values have been set, country borders have been created, and citizens have been assigned to their provinces, the series of iterations starts. Each iteration five steps are taken:

1) The level of isolation for each country is updated. It is reasonable to assume, perhaps even true by definition, that democracies do not limit the communication of their citizens with foreigners. For this reason, each country that is a democracy in this model resets the level of isolation to zero. For all other countries, the level of isolation is modeled

[4] All parameters referred to in the text are assigned random values for each run, drawn from a uniform distribution. The different parameters and the range over which initial values are set can be found in table 1.

[5] The following regression equation gives an impression of the relation between these variables and the resulting number of countries: $N_{countries} = 484.3 + .1933 \times W \times H - 41.98 \times M$, with $R^2 = 0.7560$.

[6] Throughout this paper, $N_{[a,b]}(c, d)$ is a draw from a normal distribution, with mean c, standard deviation d and truncated to the interval $[a, b]$.

Parameter	Description	Range
W	Field width	10 - 30
H	Field height	10 - 30
M	Border multiplier	0 - 20
ϕ_{mean}	Isolation mean	0 - 100
ϕ_{std}	Isolation standard deviation	1 - 30
π	Initial proportion of democratic countries	0 - 0.2
C_{mean}	Number of citizens, mean	20 - 100
C_{std}	Number of citizens, standard deviation	1 - 40
λ	Number of levels on attitude scale	3 - 300
τ	Chance of cross-border communication	0 - 1
A_{mean}	Initial attitude mean	0 - 100
A_{std}	Initial attitude standard deviation	1 - 50
U_{mean}	In-group threshold, mean	0 - 80
U_{std}	In-group threshold, standard deviation	1 - 20
T_{mean}	Out-group threshold, mean	20 - 100
T_{std}	Out-group threshold, standard deviation	1 - 20
B	Size of the effect of broadcasts	1 - λ
K	Random chance of coups	0 - 0.0002
D	Regime delay	50 - 100
β	Starting point of decaying regime fragility	0 - 0.2
γ	Strength of regime fragility decay	0 - -0.33

Table 1: Model parameters and value ranges

as a straightforward random walk:

$$\varphi_{t+1} = \begin{cases} 0 & \text{if} \quad \Omega = 1 \\ \varphi_t + U | U \in \{-1, 0, 1\} & \text{otherwise,} \end{cases} \tag{7}$$

whereby φ_{t+1} is truncated to $[0, 100]$. With a random walk it is implicitly assumed that levels of suppression generally change over time, and that without other predictive variables, the best prediction of the level of isolation at time t is likely to be the level of isolation at time $t-1$.

2) $N_{citizens}/10$ times a random citizen (S) is selected to initiate communication. The probability for each of the four provinces P in the Neumann neighborhood that a citizen will be targeted from this province is:

$$Pr(P) = \begin{cases} \tau/4 & \text{if} \quad C_P = C_S \\ \frac{\tau \max(\varphi_{C_S}, \varphi_{C_R})}{400} & \text{otherwise.} \end{cases} \tag{8}$$

The maximum level of isolation between the two countries is taken, as it is assumed that what really matters for communication to occur is whether the more restricted of the two countries can be reached.[7] If a neighboring province is selected, a citizen (R) will be randomly selected from this province, otherwise this will be done from the province of

[7] τ is divided by four because there are four neighbours in the Van Neumann neighbourhood. The division by 100 is due to the fact that φ is scaled from 0 to 100.

S. Once a sending (S) and a receiving (R) citizen have been selected, given that their attitudes towards democracy differ, the attitude of R is updated in line with the social judgment model of communication:[8]

$$\alpha_R = \begin{cases} \alpha_R + sign(\alpha_S - \alpha_R) \times 1 & \text{if} \quad |\alpha_S - \alpha_R| < u_R \\ \alpha_R - sign(\alpha_S - \alpha_R) \times 1 & \text{if} \quad |\alpha_S - \alpha_R| > t_R \\ \alpha_R & \text{otherwise,} \end{cases} \tag{9}$$

whereby α_R is truncated to $[0, \lambda - 1]$.

3) After the order in which citizens are being processed has been randomized, each citizen determines whether or not to start or stop protesting. In line with the cascading model of revolution as described above, a citizen will join the protest if the attitude against the current regime is strong enough relative to the proportion of protesters in the citizen's province to dare to risk the costs of protesting.

$$\psi_i = \begin{cases} 1 & \text{if} \quad \Omega_{C_i} = 0 \quad \& \quad \Upsilon \leq \frac{\alpha_i}{\lambda} \quad \text{or} \\ & \text{if} \quad \Omega_{C_i} = 1 \quad \& \quad \Upsilon \geq \frac{\alpha_i}{\lambda} \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

where

$$\Upsilon = \frac{\sum_{j \in P_i} \psi_j}{N_{citizens, P_i}}. \tag{11}$$

It should be pointed out that protesting is here used in a very general sense. It could be seen as people joining actual protests in the streets, but other forms of anti-regime behavior can well be imagined, for example voting for a candidate with a program to establish a strong leadership instead of a democracy in democratic elections. Protesting includes all forms of relatively public manifestations of anti-regime behavior.

4) One randomly selected democratic capital will broadcast its democratic values to citizens in neighboring provinces. In this case not the Neumann neighborhood is taken into account, but all nine provinces that are either in the Neumann neighborhood or diagonally adjacent, including the capital itself. For each of the nine provinces, the probability of receiving the broadcast is one when the province is part of the same country, or one minus the maximum level of isolation of the two countries involved. For a province that receives the broadcast, all citizens update their attitude towards democracy by the size of B. Similar to protesting in the model, this broadcasting effect should be considered a very strong abstraction of all types of attempts by a democratic country to influence opinions towards democracy in neighboring countries, ranging from radio broadcasts like *Radio Free Europe* to financing foreign pressure groups.

5) Each country determines whether or not a revolution will take place:

$$\Omega_{t+1} = \begin{cases} 1 - \Omega_t & \text{if} \quad \Upsilon_{capital} = 1 \quad \& \quad D \geq s \quad \text{or} \\ & \text{with probability} \max(K, \beta e^{\gamma s}) \\ \Omega_t & \text{otherwise,} \end{cases} \tag{12}$$

where s is the time since the last revolution or coup. There are two ways in which a country can make a transition from or towards democracy: a country makes a transition

[8] Note that this is a slight deviation from the model by Jager and Amblard (2005), since the magnitude of change in attitude is always zero or one and independent of the distance between the attitudes of the two agents.

when all citizens in the capital province are protesting[9] or randomly, with a probability which decays with the age of the regime. The latter can be seen as external shocks to the model, the many forms of revolutions in the world that are not caused by the diffusion of attitudes or even by public opinion in the first place. When Gorbachev let the Soviet Union slip and the regime collapsed, this can hardly be seen as an effect of democratic diffusion, but the subsequent collapses of many regimes in Eastern Europe is to some extent explained by this factor. The external shock of Gorbachev thus generated a diffusion effect as it is modeled here. Another way to look at this would be to see the K as the built-in error in the model, much like any econometric model will include an error term. The fact that for endogenous revolutions only the capital is taken into account can be defended by a quick glance at most coups and revolutions in the world. Protests are generally more threatening when they take place in the capital, and rarely can a country where all but one region are opposed to the regime sustain its political system. Taking into account all provinces leads to unrealistic assumptions, while taking the capital into account seems in line with general perceptions of revolutions. Finally, for a number of iterations after a revolution, set by D, revolutions are not possible.

5 Simulation results

An agent-based model is to a considerable extent random: parameters are initialized with random values within certain relatively arbitrary preset ranges[10] and parts of the model that are considered exogenous are implemented as random events, like the stochastic element in the random walk of the level of isolation for non-democracies and the random chance of coups independent of public opinion. Similar to statistical analysis, the solution to separate random effects from modeled ones lies to a large extent in the size of the 'sample'. Whereas we can get more accurate estimates of model parameters in statistics by taking larger random samples, in agent-based modeling we acquire a similar large N by running the model many times. For this paper, the model has been run 2481 times. Each of these runs generates a dataset with information on every hundredth iteration, and some overall statistics describing the run as a whole, including some graphical time trends. These results will be studied further in this section.

5.1 Iterations and time

One issue that concerns some agent-based models, including this one, is how to match iterations to time. Iterations of the model are an abstraction of time, are the time dimension in the model, but it is entirely unclear how iterations relate to the real time that is being modeled. To be able to compare time trends in the model, it is necessary to be able to make this match. One way to get around this problem is to make strong assumptions about the length of an iteration, e.g. to see them as years or months. Usually this leads to a match either because it is convenient in terms of data collection - we generally have annual data on institutional developments - or because it is a nice rounded value - days,

[9] A requirement of 100% protesters might seem too strong, but given the mechanisms of the cascading model of revolution this is theoretically the most appropriate and also in practice does not hold back many countries where there is a large proportion of protesters smaller than 100%.

[10] It is in fact more common to do parameter sweeps - iterate over a fixed set of parameter combinations. Such fixed sets of parameters, however, are relatively arbitrary in nature and make it more difficult to find critical values and see the precise relationship between the parameters and the model output. In this paper, the parameter settings are seen as a random sample from the overal parameter space.

months, years. There is no reason, however, to make this kind of assumption. For the analysis below I have avoided to make any such assumption by simply taking the iteration length that best fits the data, for each run separately. The scaling parameters ξ is estimated using:

$$D_{model,t} = D_{data,\xi t+\nu} + \varepsilon, \tag{13}$$

whereby ν is fixed to 1810 and $\frac{\varepsilon^2}{n}$ is minimized to get the optimal scaling.[11] The subscript t refers to the number of the iteration. This minimization is simply done iteratively, iterating over the range: $.023 \leq \xi \leq .047$.[12] Once the optimal ξ is found for each run, this parameter is used to scale the empirical data in all analyses below.

5.2 Waves in the model

As discussed above, empirical data show a clear pattern of waves. To what extent are similar waves observable in the simulation results? Waves have been measured as the variance of the proportion of democracies over time in the simulation output around a moving average of this trend. The trend itself was smoothened by a moving average with a window of 210 iterations to avoid including very small waves and the variance was then taken around a moving average of 2010 iteretions. An attempt to analyze the relation between the model parameters and the presence of global waves is presented in table 2, which is the result of a ordinary least squares regression explaining $\log(waves)$, using standardized variables. A number of parameters are clearly related to the presence of waves in the simulation output. The refinement of the measurement of attitudes towards democracy is negatively related to the presence of waves. When we define the attitudes on a more refined scale in the simulation, the changes become more smooth and the presence of waves less likely. The multiplier, which is directly related to the number of countries in the model (see footnote 5), is also negatively related to waves. The larger the countries, the less likely waves become. In terms of initial values, the proportion of democracies at the start of the simulation is also negatively related to waves, as well as the initial average attitude towards democracy.

5.3 A real-world outcome?

We will now turn to the conditions under which the simulations shws output that is similar to the empirically observed data. One would be tempted to look first at the correlation between the model data and the scaled empirical data, but the problem of such a measure is that a model where the waves take place at very different points in time than in the real data is not necessarily a bad match. The measure used to scale the empirical data to the model iterations, the minimization of $\frac{\varepsilon^2}{n}$, can also be used to find the models that lead to a relative number of democracies, over time, similar to the empirical data. Figures 6 and 7 show the extremes of this measure. In table 3 an analysis is presented of a linear model with the log of ε^2 as the dependent variable and a selection of the model parameters as standardized explanatory variables. The level of refinement of the measurement of attitudes towards democracy, λ, is negatively related to ε^2. A more refined measurement thus leads to a model that better matches the empirical data. The second parameter

[11] The size of n, the number of iterations for which there is a matching data point in the Vanhanen data set, varies with the scaling parameter ξ and thus has to be taken into account when minimizing ε^2.

[12] These values are selected to limit the scaling between using the full simulation of 8000 iterations as a match to real world data and using about half of this range.

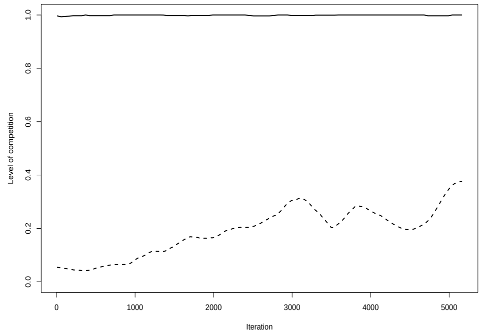

Figure 4: Absence of Waves of Democracy

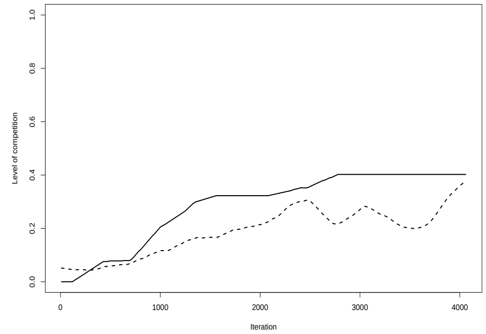

Figure 5: Presence of Waves of Democracy

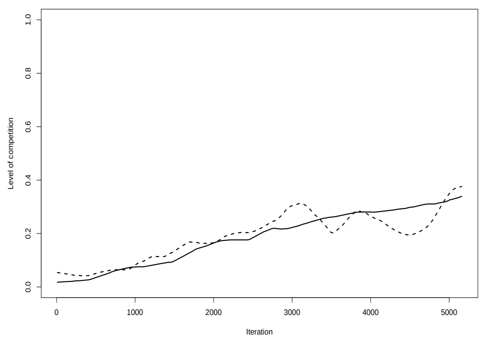

Figure 6: Similar pattern (low ε^2)

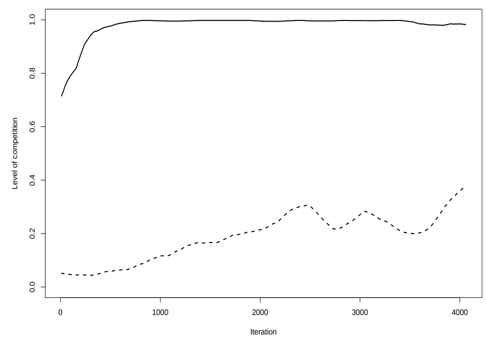

Figure 7: Dissimilar pattern (high ε^2)

showing a significant correlation is the initial average attitude towards democracy, A_{mean}, which has a positive coefficient. The higher this initial value is set in the model, the less likely the model is to reflect the empirical situation in the early nineteenth century, when democracies were relatively uncommon. The 'error parameter' of the model, the random chance of coups independent of the attitudes of the citizens, K, is somewhat related to the match as well. A higher probability of random coups makes the model less likely to match. The relation between ε^2 and the strength of the decay of regime fragility, γ, is negative, implying that the faster this temporary fragility after a revolution or coup decays, the less likely the model is to match the empirical data. The combination of a low random chance of revolution and a fast decay of initial fragility suggests that a model with quick and frequent shocks to the system is relatively unlikely to resemble the empirically observed patterns. The last parameter of relevance in this matching exercise is the border multiplier M, which is directly related to the sizes of the countries in terms of the number of provinces they include. A high multiplier means more 'conquers' in the border generation algorithm. The bigger the countries in this model, the more likely the model is thus to fit the empirical data.

In terms of the geographical clustering, one of the key empirical phenomena that this paper attempts to explain, a simple measure has been used to estimate the applicability of the model to the real world data. The clustering coefficient, Moran's I (I_t), of the

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-6.0302	0.2383	-25.31	0.0000
Number of levels (λ)	-0.0018	0.0006	-2.79	0.0055
Initial proportion democracies (π)	-1.1774	0.4990	-2.36	0.0187
Initial average attitude (A_{mean})	0.0009	0.0011	0.79	0.4300
Initial average isolation (ϕ_{mean})	-0.0024	0.0010	-2.38	0.0179
Initial average nr of citizens (C_{mean})	-0.0002	0.0012	-0.15	0.8838
Regime delay (D)	0.0015	0.0019	0.78	0.4379
Chance of cross-border communication (τ)	-0.0583	0.0927	-0.63	0.5295
Random chance of coups (K)	5120.1753	4905.2328	1.04	0.2970
Average in-group threshold (U_{mean})	-0.0008	0.0013	-0.65	0.5190
Average out-group threshold (T_{mean})	0.0011	0.0012	0.91	0.3639
Broadcast effect (B)	0.0013	0.0008	1.61	0.1075
Strength of regime fragility decay (γ)	-0.6250	0.3070	-2.04	0.0423
Border multiplier (M)	-0.0456	0.0051	-8.91	0.0000
Field size ($W \times H$)	-0.0001	0.0002	-0.78	0.4355

Table 2: Explanatory model parameters on the level of waves ($\log(waves)$)

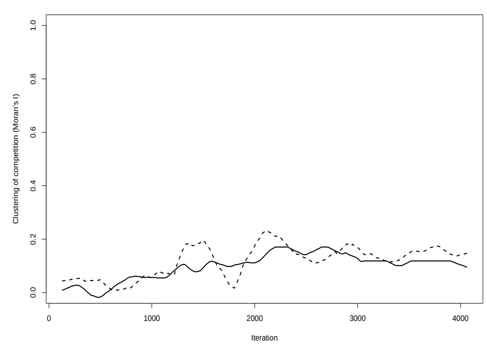

Figure 8: Similar pattern of clustering

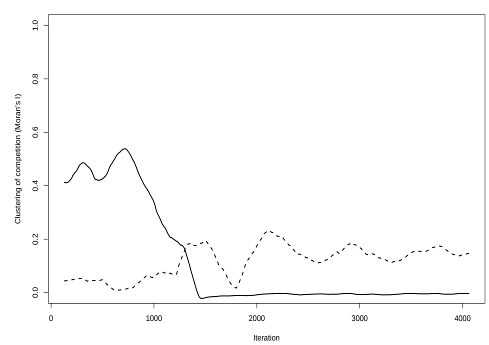

Figure 9: Dissimilar pattern of clustering

simulation is matched onto the empirical data on clustering, in a similar fashion to the way the proportion of democracies has been matched, using the previously estimated ξ as scaling parameter. The best match in the data is displayed in figure 8 and the worst match in figure 9. A regression analysis similar to above for the level of waves and the match of the overal pattern, the two parameters that stand out are M, the multiplier, and $W \times H$, the size of the map.[13] The larger the grid of the simulation and the higher the multiplier, thus the larger the countries in terms of provinces, the more likely the pattern of clustering is to resemble the empirically observed clustering.

[13] When all countries are either democratic or non-democratic, Moran's I cannot be calculated ($\sum_i (x_{it} - \overline{x}_t)^2$, part of the nominator, will be zero). This leads to problems when calculating the extent to which the two trends are similar. For this reason, all simulations where this was the case for more than 300 iterations were removed from the regression, thus dropping 512 out of 2481 cases.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	−0.5986	0.0729	−8.21	0.0000
Number of levels (λ)	−0.4750	0.0910	−5.22	0.0000
Initial proportion democracies (π)	0.0003	0.0376	0.01	0.9941
Initial average attitude (A_{mean})	0.3746	0.0444	8.43	0.0000
Initial average isolation (ϕ_{mean})	−0.0450	0.0369	−1.22	0.2228
Initial average nr of citizens (C_{mean})	0.0139	0.0374	0.37	0.7105
Regime delay (D)	0.0620	0.0373	1.66	0.0970
Chance of cross-border communication (τ)	0.0126	0.0366	0.34	0.7302
Random chance of coups (K)	0.0893	0.0371	2.40	0.0165
Average in-group threshold (U_{mean})	0.0731	0.0379	1.93	0.0539
Average out-group threshold (T_{mean})	0.0132	0.0373	0.35	0.7242
Broadcast effect (B)	0.0020	0.0913	0.02	0.9823
Strength of regime fragility decay (γ)	−0.1440	0.0379	−3.80	0.0002
Border multiplier (M)	−0.1670	0.0390	−4.28	0.0000
Field size ($W \times H$)	−0.0193	0.0354	−0.55	0.5855

Table 3: Explanatory model parameters on $\log(\varepsilon^2)$

6 Conclusion

The question asked at the start of this paper is an attempt to find under what conditions diffusion of democracy through popular opinion can lead to the waves, time trends, and geographical clustering as can be observed in the past two centuries. The relationship between the patterns of interest and the position in the large parameter space was studied for a large number of simulations using an agent-based model with as prime building blocks the cascading model of revolution or spiral of silence and the social judgment model of interpersonal communication. In an artificial world of provinces, countries, and citizens with particular attitudes, behaviors and propensities to accept the viewpoint of other citizens, patterns were visible similar to those observed in empirical data covering the past two centuries, given particular parameter settings.

Some of the parameters important in their effects on the similarity between the model output and the empirical data are relatively technical in nature. For example, the refinement of the measurement of the attitude towards democracy of each agent in the model is of crucial importance, but is difficult to translate into an observable variable in empirical research. Other parameters that are relevant are more interesting substantively, however. The parameter that governs the size of the countries in the model, in terms of the number of provinces each country encompasses, is significantly and positively related to the existence of waves, overall trend, and levels of clustering similar to the empirical data. Initial values in terms of the proportion of democracies and the average initial attitude towards democracy are also significant parameters. Although substantively interpretable, their importance is not surprising. We know that the world in 1810 was relatively undemocratic, and any simulation that start otherwise is unlikely to match empirical patterns.

The simulation results suggest that it is possible the observe patterns of democratic growth, international waves of democratization and international clustering of democracies with a model based on the idea of cascading revolutions, a simple model of communication between citizens, and the existince of democratic propaganda, the attempt of democratic countries to affect outcomes in neighboring countries. The model results are a good indication of the theoretical implications of the model and suggest some internal consistency and validity to the model, but external, empirical validity has not been established in this paper and will be left to later work.

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